

A method to measure the antikaon-nucleon scattering length in lattice QCD ^{#1}

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Abstract

We propose a method to determine the isoscalar $\bar{K}N$ scattering length on the lattice. Our method represents the generalization of Lüscher's approach in the presence of inelastic channels (complex scattering length). In addition, the proposed approach allows one to find the position of the S-matrix pole corresponding to the $\Lambda(1405)$ resonance.

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1. The antikaon-nucleon scattering amplitude is of fundamental importance in nuclear, particle and astrophysics, see e.g. Refs. [1–4]. In particular, the $\bar{K}N$ system at threshold provides an interesting testing ground of the chiral dynamics of QCD with strange quarks due to the $\Lambda(1405)$ resonance just below the scattering threshold. In fact, experimental information on the K^-p scattering length from scattering data and kaonic hydrogen level shifts is contradictory, as first stressed in [5] and further elaborated on in Refs. [6–8]. A clarification is expected from the upcoming SIDDHARTA experiment at DAΦNE, that intends to remeasure kaonic hydrogen with unprecedented accuracy and is expected to give further constraints to the isoscalar and isovector kaon-nucleon scattering lengths from the first measurement of the energy spectrum of kaonic deuterium.

On the theoretical side, effective field theory methods in various disguises are employed to pin down the $\bar{K}N$ scattering length. The scattering information is usually analyzed in terms of unitarized versions of chiral perturbation theory, that lead to a dynamic generation of various resonances, in particular the $\Lambda(1405)$. Such schemes have been worked out over the years by various groups at various levels of sophistication (see, e.g. [6–9]). While an impressive amount of data (cross sections, threshold ratios, mass distributions, etc.) is described in such approaches with good precision, unitarization of course introduces some unwanted model-dependence. On the other hand, the extraction of the $\bar{K}N$ scattering length from kaonic hydrogen is firmly rooted in non-relativistic bound-state effective field theory and thus is devoid of the above-mentioned model-dependence (for a recent review, see [10]). However, it entirely rests on the availability of precise kaonic atom data. Unfortunately, the existing data from DEAR [11] and KEK [12] are conflicting. It would therefore be most welcome to have another tool at hand that would allow one to determine this fundamental quantity.

As we will argue in this letter, lattice QCD provides such a framework. As first shown by Lüscher, finite volume simulations of the energy levels of two-particle states can give access to scattering information [13, 14]. The idea of Lüscher is very elegant and simple. For two-particle states that are well separated from bound states or resonances in the given channel, the $1/L$ expansion of the energy levels takes the generic form (here, L is the size of the box with volume $L \times L \times L$)

$$E \sim \frac{a}{L^3} \left\{ 1 + c_1 \frac{a}{L} + c_2 \frac{a^2}{L^2} \right\} + \mathcal{O}(L^{-6}) , \quad (1)$$

where a is the desired scattering length and c_1, c_2 are pure numbers (see below). The method has e.g. been used to extract the $\pi\pi$, πK and KN S -wave scattering lengths from lattice data [15]. Note that an alternative proposal to extract the scattering length from the two-particle wave function is e.g. given in Ref. [16].

However, for the extraction of the $\bar{K}N$ scattering length, a generalization of this scheme is called for since there is a strong channel coupling between $\bar{K}N$ and $\Sigma\pi$, the latter channel having its threshold about 100 MeV below the opening of the $\bar{K}N$ one. In addition, the appearance of the $\Lambda(1405)$ just between these two thresholds further complicates the picture. All these features can be captured by a two-channel Lippmann-Schwinger equation. As we will show in the following, a suitable formulation of this equation in the finite volume allows for an unambiguous extraction of the complex-valued isoscalar K^-N scattering length. Note also that our method is similar to the approach adopted in Ref. [17], where the problem was treated within the potential scattering theory. In this paper, we use instead the language of the non-relativistic effective field theory (NR EFT), which enables one to systematically address the effects of particle creation/annihilation and relativistic corrections.

2. To set the stage, we consider a two-channel Lippmann-Schwinger (LS) equation in NR EFT in the infinite volume. Note that we are using a covariant version of the NR EFT, considered in Refs. [18]. The channel number 1 refers to $\bar{K}N$ and 2 to $\Sigma\pi$ with total isospin $I = 0$. The resonance $\Lambda(1405)$ is

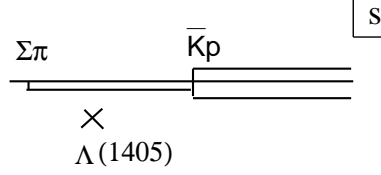


Figure 1: Complex s -plane with the $\Sigma\pi$ and $\bar{K}N$ cuts along the real axis and the location of the $\Lambda(1405)$ resonance.

located between two thresholds, on the second Riemann sheet, close to the real axis (these thresholds are defined by $s_t = (m_N + M_K)^2$ and $s'_t = (m_\Sigma + M_\pi)^2$)^{#5}. Consider first energies above $\bar{K}N$ threshold, $s > (m_N + M_K)^2$. The coupled-channel LS equation for the T -matrix elements $T_{ij}(s)$ in the dimensionally regularized NR EFT reads (we only consider S -waves here)

$$\begin{aligned} T_{11} &= H_{11} + H_{11} i q_1 T_{11} + H_{12} i q_2 T_{21} , \\ T_{21} &= H_{21} + H_{21} i q_1 T_{11} + H_{22} i q_2 T_{21} , \end{aligned} \quad (2)$$

with $q_1 = \lambda^{1/2}(s, m_N^2, M_K^2)/(2\sqrt{s})$, $q_2 = \lambda^{1/2}(s, m_\Sigma^2, M_\pi^2)/(2\sqrt{s})$ and $\lambda(x, y, z)$ stands for the Källen function. Furthermore, the $H_{ij}(s)$ denote the driving potential in the corresponding channel. Continuation of the center-of-mass momentum q_1 below threshold $(m_\Sigma + M_\pi)^2 < s < (m_N + M_K)^2$ is obtained via (see Fig. 1 for the corresponding analytical structure)

$$i q_1 \rightarrow -\kappa_1 = -\frac{(-\lambda(s, M_K^2, m_N^2))^{1/2}}{2\sqrt{s}} \quad (3)$$

The resonance corresponds to a pole on the second Riemann sheet in the complex s -plane, its position can be determined from the secular equation

$$\Delta(s) = 1 + \kappa_1^R H_{11} - \kappa_2^R H_{22} - \kappa_1^R \kappa_2^R (H_{11} H_{22} - H_{12}^2) \quad (4)$$

with $\kappa_1^R = -(-\lambda(s_R, m_N^2, M_K^2))^{1/2}/(2\sqrt{s_R})$ and $\kappa_2^R = (-\lambda(s_R, m_\Sigma^2, M_\pi^2))^{1/2}/(2\sqrt{s_R})$. The energy and width of the resonance are then given by $\sqrt{s_R} = E_R - i\Gamma_R/2$. The $\bar{K}N$ scattering length is related to the amplitude T_{11} at $s = s_t = (m_N + M_\pi)^2$ via

$$a_{11} \equiv T_{11}(s_t) = H_{11}(s_t) + \frac{i q_2(s_t) (H_{12}(s_t))^2}{1 - i q_2(s_t) H_{22}(s_t)} . \quad (5)$$

Thus, to pin down its complex value, we need to determine the three real quantities H_{11}, H_{12}, H_{22} at $s = s_t$ appearing in Eq. (5)

3. We now consider the same problem in a finite volume. The rotational symmetry is broken to a cubic symmetry so that the infinite volume version of the LS equation Eq. (2) takes the form (we consider only S -waves here, neglecting the small mixing to higher partial waves. The mixing can be easily included at latter stage, see e.g., Ref. [19].),

$$\begin{aligned} T_{11} &= H_{11} - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_1^2) H_{11} T_{11} - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_2^2) H_{12} T_{21} , \\ T_{21} &= H_{21} - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_1^2) H_{21} T_{11} - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_2^2) H_{22} T_{21} , \end{aligned} \quad (6)$$

^{#5}In the following, we work in the isospin limit and thus do not resolve the further splitting of these thresholds.

with

$$\begin{aligned}
k_1^2 &= \left(\frac{L}{2\pi}\right)^2 \frac{\lambda(s, M_K^2, m_N^2)}{4s}, \\
k_2^2 &= \left(\frac{L}{2\pi}\right)^2 \frac{\lambda(s, M_\pi^2, m_\Sigma^2)}{4s}, \\
Z_{00}(1; k^2) &= \frac{1}{\sqrt{4\pi}} \lim_{r \rightarrow 1} \sum_{\vec{n} \in R^3} \frac{1}{(\vec{n}^2 - k^2)^r}.
\end{aligned} \tag{7}$$

Here, we have neglected the terms that vanish exponentially at a large L . The secular equation that determines the spectrum can be brought into the form

$$\begin{aligned}
1 - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_2^2) F(s, L) &= 0, \\
F(s, L) &= \left[H_{22} - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_1^2) (H_{11}H_{22} - H_{12}^2) \right] \left[1 - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_1^2) H_{11} \right]^{-1}
\end{aligned} \tag{8}$$

This is rewritten as

$$\begin{aligned}
\delta(s, L) &= -\phi(k_2) + n\pi, \quad n = 0, 1, 2, \dots \\
\phi(k_2) &= -\arctan \frac{\pi^{3/2} k_2}{Z_{00}(1; k_2^2)},
\end{aligned} \tag{9}$$

with

$$\tan \delta(s, L) = q_2(s) F(s, L). \tag{10}$$

$\delta(s, L)$ is called the *pseudophase*. It is a function of the energy \sqrt{s} and the level index n , $\delta_n(s) = \delta(s, L_n(s))$.

The dependence of the pseudophase on s and L (or, equivalently, on the level index n) is very different from that of the usual scattering phase. Namely, the elastic phase extracted from the lattice data by using Lüscher's formula is independent of the volume modulo terms that exponentially vanish at a large L . Further, the energies where the phase passes through $\pi/2$ lie close to the real resonance locations. In contrast with this, the pseudophase contains terms which are only power suppressed at a large L . Moreover, it contains the tower of resonances which are not related to the dynamics of the system in the infinite volume and merely reflect the existence of the discrete energy levels in the “shielded” channel.

Measuring the pseudophase on the lattice can be used to determine the $\bar{K}N$ scattering length. It can be directly seen from the expression of the pseudophase, which depends on real functions H_{11}, H_{12}, H_{22} . Extracting these from the data, we then find the scattering length by using Eq. (5). Note that in the expression for the scattering length we need $H_{ij}(s)$ evaluated at threshold $s = s_t$. We shall however demonstrate below that replacing $H_{ij}(s)$ by $H_{ij}(s_t)$ in certain observables, related to the pseudophase, introduces very small correction, since the effective range term proportional to $(s - s_t)$ is suppressed by L^{-3} as compared to the leading order result. To be specific, we consider the following three observables:

1. For some chosen value of n , we measure the value of the pseudophase $\delta(s_t; L(s_t)) \doteq \delta_t$ at threshold $s_t = (m_N + M_K)^2$ and $E_t = \sqrt{s_t}$ (see Fig. 3). On the other hand, we may express δ_t through H_{ij} at $s = s_t$ in the following way. At threshold, Z_{00} is singular,

$$Z_{00}(1; k^2) = -\frac{1}{\sqrt{4\pi}} \frac{1}{k^2} + \mathcal{O}(1), \tag{11}$$

so that

$$F(s, L)|_{s \rightarrow s_t} = H_{22}(s_t) - H_{12}^2(s_t)/H_{11}(s_t) \quad (12)$$

$$\tan \delta(s_t; L(s_t)) = q_2(s_t) (H_{22}(s_t) - H_{12}^2(s_t)/H_{11}(s_t)) \doteq q_2(s_t) I(s_t) . \quad (13)$$

Thus, measuring δ_t , we may extract the combination $H_{22} - H_{12}^2/H_{11}$.

2. Suppose that $\tan \delta(s; L(s))$ is infinite at $s = s_3 = E_3^2$ and $L = L_3 = L(s_3)$ (see Fig. 2 for a specific representation of the pseudophase based on a two-channel K-matrix model described below). This occurs at the energy where the denominator of Eq. (8) vanishes

$$1 - \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_1^2(s_3)) H_{11}(s_3) = 0 . \quad (14)$$

We solve this equation by expanding both $H_{11}(s)$ and $Z_{00}(1; k_1^2(s))$ in Taylor series in the vicinity of $s = s_t$

$$H_{11}(s) = H_{11}(s_t) + q_1^2(s) H'_{11}(s_t) + \mathcal{O}(q^4) \quad (15)$$

and

$$\frac{2}{\sqrt{\pi}L} Z_{00}(1; k^2) = \frac{1}{\pi L} \frac{1}{k^2} + \frac{c_1}{L} + \frac{\pi k^2}{L} (c_1^2 - c_2) + \mathcal{O}(k^4) , \quad (16)$$

with

$$c_1 = \frac{1}{\pi} \lim_{r \rightarrow 1} \sum_{\mathbf{n} \neq 0} \frac{1}{(\mathbf{n}^2)^r} = -2.837297 \dots , \quad c_2 = c_1^2 - \frac{1}{\pi^2} \sum_{\mathbf{n} \neq 0} \frac{1}{\mathbf{n}^4} = 6.375183 \dots . \quad (17)$$

Substituting Eqs. (15) and (16) into Eq. (14), we obtain

$$q_1^2(s_3) = -\frac{4\pi H_{11}(s_t)}{L_3^3} \left(1 + c_1 \frac{H_{11}(s_t)}{L_3} + c_2 \frac{H_{11}(s_t)^2}{L_3^2} + \mathcal{O}\left(\frac{1}{L_3^3}\right) \right) . \quad (18)$$

This means that measuring the value of s , where the $\tan \delta(s; L(s))$ becomes infinite, we may extract H_{11} at $s = s_t$. Note that the effective-range term, which contains $H'_{11}(s_t)$, contributes first at $\mathcal{O}(L^{-6})$.

3. Similarly, suppose that $\tan \delta(s, L) = 0$ at $s = s_2 = E_2^2$ and $L = L_2 = L(s_2)$ (see Fig. 2). Using the same technique as just described, we obtain:

$$q_1^2(s_2) = -\frac{4\pi G(s_t)}{L_2^3} \left(1 + c_1 \frac{G(s_t)}{L_2} + c_2 \frac{G(s_t)^2}{L_2^2} + \mathcal{O}\left(\frac{1}{L_2^3}\right) \right) , \quad (19)$$

where

$$G(s_t) = H_{11}(s_t) - H_{12}^2(s_t)/H_{22}(s_t) . \quad (20)$$

Therefore, measuring s_2 , we get $H_{11} - H_{12}^2/H_{22}$.

Using finally Eq. (5), we can express the scattering length in terms of the three quantities H_{11} , I and G , all taken at $s = s_t$

$$a_{11} = H_{11}(s_t) + \frac{i q_2(s_t) I(s_t) H_{11}(s_t) (H_{11}(s_t)/G(s_t) - 1)}{1 - i q_2(s_t) I(s_t) H_{11}(s_t)/G(s_t)} . \quad (21)$$

This is the central result of this letter.

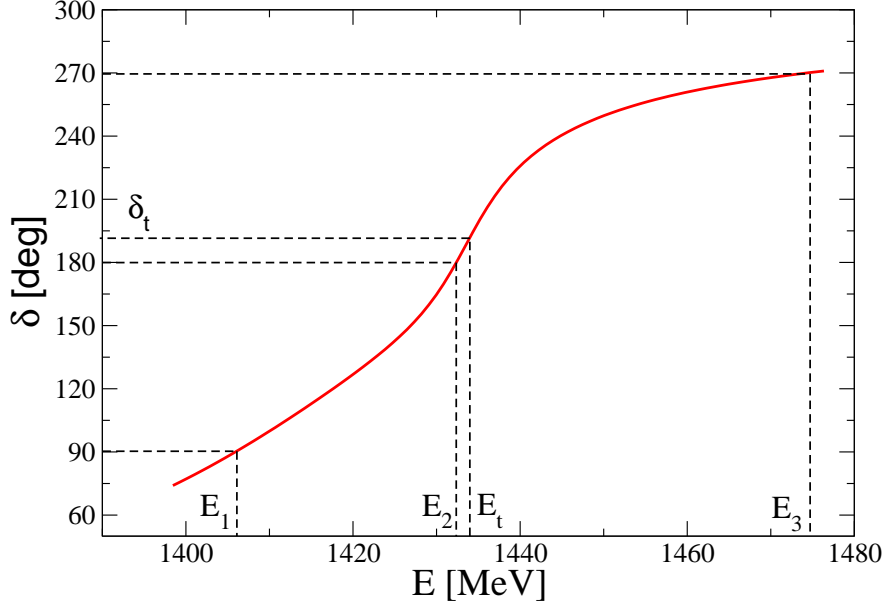


Figure 2: The pseudophase δ . The energy E_1 at which the pseudophase passes through $\pi/2$ corresponds to the $\Lambda(1405)$.

Finally, note that in the analysis of the lattice data it may be more convenient to directly fit the explicit expression of the pseudophase given in Eq. (8) to the measured values on the lattice around $s = s_t$, replacing $H_{ij}(s)$ by $H_{ij}(s_t)$ and considering $H_{ij}(s_t)$ ($ij = 11, 12, 22$) as three independent fitting parameters. From the above discussion one may expect that such a fit will lead to the precise determination of these parameters. The effective range terms can be neglected since their contribution is suppressed by three powers of L . In this case, one does not need to measure the pseudophase in the whole interval between s_2 and s_3 .

4. Given the parameters H_{ij} determined from fitting to the pseudophase, the position of the pole on the second Riemann sheet of the complex variable s , which corresponds to the $\Lambda(1405)$ -resonance, can be determined from the secular equation (4). We expect that replacing $H_{ij}(s)$ by $H_{ij}(s_t)$ allows one to locate the pole position at a reasonable accuracy.

5. In order to demonstrate the above-described proposal in practice, we have investigated a coupled-channel model with an explicit $\Lambda(1405)$ resonance located at $\text{Re } \sqrt{s_R} = 1406 \text{ MeV}$ and $-2 \text{Im } \sqrt{s_R} = 50 \text{ MeV}$. Effective range terms are neglected. The matrix elements H_{ij} are taken equal to

$$H_{11} = -1.47573 \text{ fm}, \quad H_{12} = 0.91581 \text{ fm}, \quad H_{22} = -0.34159 \text{ fm}. \quad (22)$$

This corresponds to $a_{11} = a_0(K^-N) = (-1.26 + i0.70) \text{ fm}$. The resulting first four energy levels as a function of $M_\pi L$ are shown in Fig. 3. We notice that the lowest level ($n = 1$) does only show a moderate volume dependence in the interval considered, quite in contrast to the excited ones with $n \geq 2$. For $M_\pi L \simeq 2 \dots 3$ the ground state level flattens around $E = 1406 \text{ MeV}$ that corresponds to the $\Lambda(1405)$. It is clear that, for this reason, the lowest level can not be used for the extraction of the $\bar{K}N$ scattering length. The excited levels show a more complicated behavior in this interval of L . At the first glance, these levels exhibit the so-called avoided level crossing somewhere between 1430 MeV and 1440 MeV. In the elastic case, such a behavior of the energy levels signalizes the presence of a narrow resonance near this energy. However, this is not the case here. The peculiar behavior of the

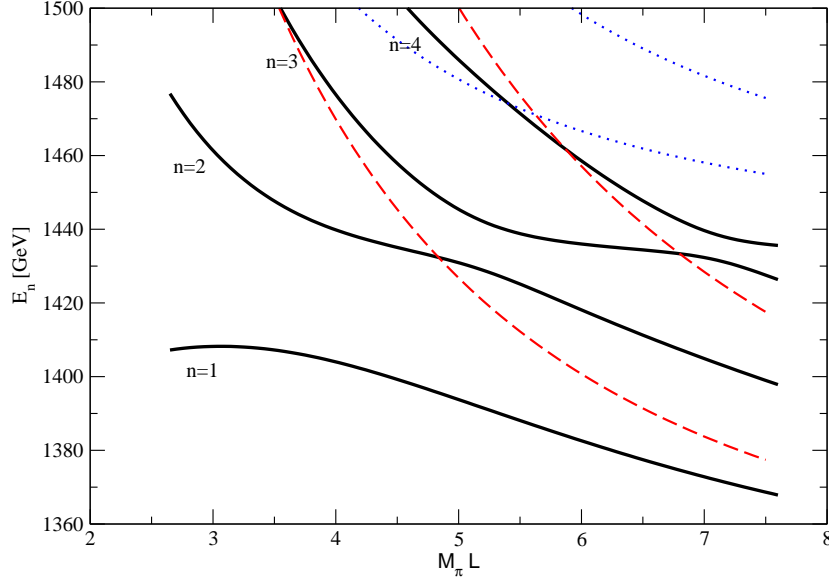


Figure 3: Energy levels for the two-channel model with an explicit $\Lambda(1405)$ resonance in the finite volume. The avoided level crossing which is observed at the energies between 1430 MeV and 1440 MeV is not related to the physical resonance in the infinite volume but reflects the presence of the $\bar{K}N$ threshold. For comparison, we plot the energy levels for the non-interacting two-particle systems $\pi\Sigma$ (dashed lines) and $\bar{K}N$ (dotted lines).

excited energy levels is caused by the opening of the $\bar{K}N$ threshold. At higher energies, the picture repeats – an avoided level crossing emerges, if the $\bar{K}N$ system has a discrete eigenvalue at this energy in a finite volume. If the volume changes, the avoided level crossing moves (in difference to the avoided level crossing corresponding to the “true” resonance). For $L \rightarrow \infty$ the bifurcation lines accumulate at threshold $s = s_t$. In this limit, the scattering amplitude is not analytic at $s = s_t$ (unitary cusp).

In Fig. 2 gives the pseudophase derived from the second ($n = 2$) energy level. It shows the expected behavior. First, it crosses $\pi/2$ at $\sqrt{s} = E_1$, very close to the mass of the $\Lambda(1405)$. Then, it passes π at $\sqrt{s} = \sqrt{s_2} = E_2$, close to the threshold where its value is $\delta_t > \pi$. At E_2 , the tangent of the pseudophase vanishes since $q_1^2(s_2) < 0$, we can conclude that $G(s_t) > 0$, cf. Eq. (19). Finally, the value of $3\pi/2$ is reached at $\sqrt{s} = \sqrt{s_3} = E_3$. Here, $q_1^2(s_3) > 0$ and consequently $H_{11}(s_t) < 0$. This can be deduced from Eq. (19) after the substitutions $s_2 \rightarrow s_3$ and $L_2 \rightarrow L_3$.

6. In this letter, we have generalized Lüscher’s algorithm for the extraction of the scattering length from the finite-volume energy spectrum measured on the lattice. The modified algorithm applies to the case when the scattering length is complex due to the presence of the open channel(s) below threshold. In the case of the $\bar{K}N$ scattering with total isospin $I = 0$, the scattering length can be determined by measuring the volume dependence of the first excited level around the threshold energy.

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